# Predicting human-structure interaction with a stable walking controller: synchrony and entrainment on the London Millennium Bridge

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**Introduction.**

In previous work, we have shown that energy optimality can be a useful qualitative predictor for the unusual walking behavior demonstrated by humans when interacting with bridges and walkways [1]. Here, we consider bipeds stabilized by feedback control walking on an externally oscillated platform as well as a passive bridge with sideways compliance. This model predicts emergence of humans walking in a manner that shakes the bridge – the synchronization of pedestrians walking on a laterally oscillating bridge depends on the number of pedestrians. Their entrainment to an externally shaken platform depends on oscillation amplitude and frequency.

**Model Details.**

We consider a simple biped with a point-mass upper body and mass-less legs. Each stride of this biped consists of two single-stance inverted pendulum motions separated by a heel-strike and a push-off impulse. The dynamics of the swing-leg are not considered and it is assumed to move to the next foot position at the end of each stance. The controller consists of a foot-position controller (as in [2]), which determines step-length and step-width based on mid-stance state of the current step, and a push-off controller, which determines push-off impulse from mid-stance state of the current step. This feedback controller stabilizes a nominal inverted pendular walking gait derived by minimizing a work-like cost for the biped in the absence of any external perturbations as reported in [1]. As in our previous work [1], we consider infinite inertia platforms (similar to shaken treadmills) and finite inertia platforms (spring-mass damper approximations of bridges and walkways).

**Methods and results.**

For the externally oscillated infinite inertia platform, we vary the frequency and amplitude of sinusoidal platform oscillations and determine how this affects pedestrian synchronization. We find that for a given pedestrian speed there is a narrow range of frequencies and amplitudes for which the pedestrians will entrain (synchronize) to the platform oscillation as shown in figure 1.

For the passive finite inertia platform (bridge), we simulate multiple pedestrians interacting with the bridge. Because we could not 'directly' simulate hundreds of pedestrians, we simulated N human (in the hundreds) by simulating p = 1, 2, 4, or 6 biped total. Each of those p simulated bipeds thus has mass equal to N/p times a single human, and represents a group of N/p humans walking in sync. The p bipeds' initial relative phase is randomized. As shown in figure 2, we find that oscillations decay or are sustained only as a function of the effective mass of all pedestrians the number of groups does not matter.

**Acknowledgements.**

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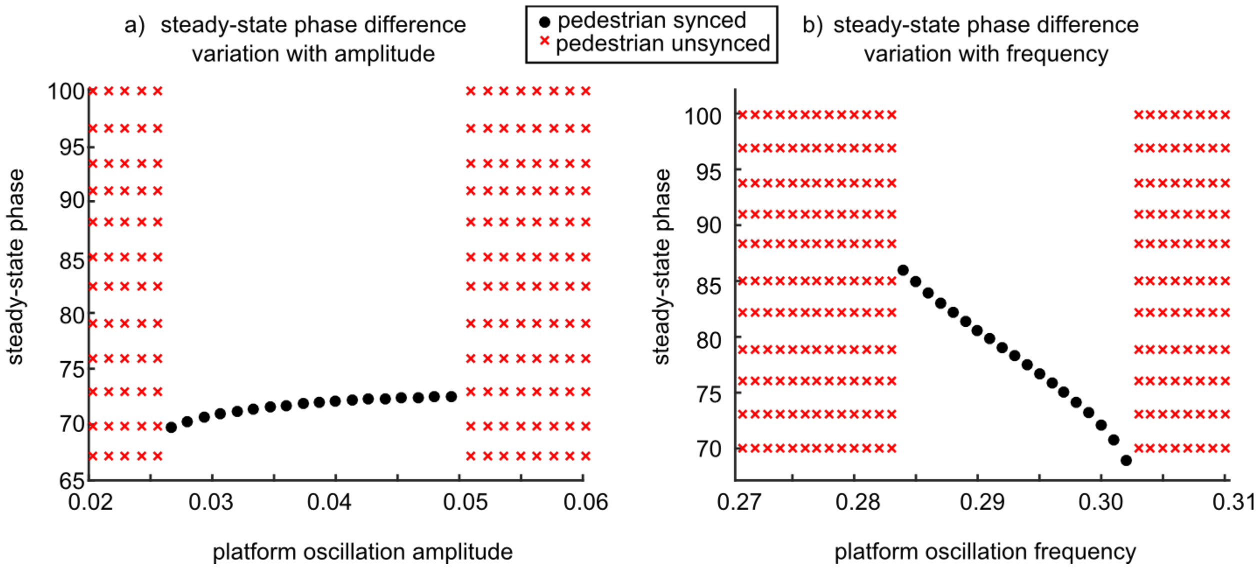


**Figure 1.** Steady-state phase difference between pedestrian and platform as a function of a) platform oscillation amplitude and b) platform oscillation frequency (all units non-dimensional). We find that pedestrians entrain to platform oscillations only for a small range of frequencies and amplitudes (black filled circles). At very high amplitudes the pedestrians fall, but in all other unsynced cases (region with red crosses) the phase difference between the pedestrian and the platform varies from step to step indicating the absence of entrainment.

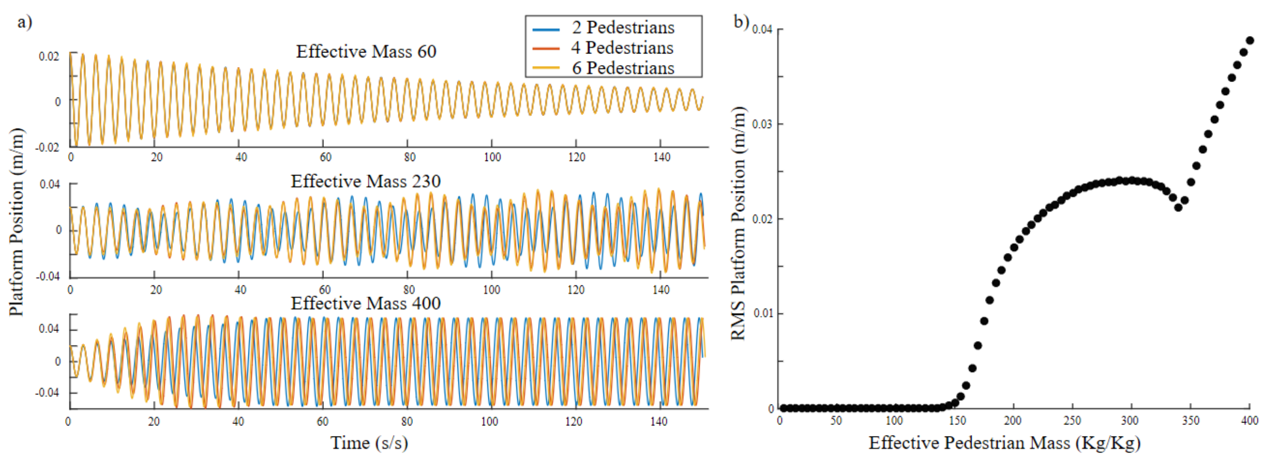


**Figure 2.** a) Platform oscillation over time as 2 (blue), 4 (red) and 6 (yellow) groups of pedestrians each with an effective total mass of 60, 320 or 400 pedestrians walks. We see that the steady-state result is independent of the number of groups considered if the effective mass is the same, barring time offsets. We see decaying oscillations for low pedestrian numbers, oscillations with multi-step periodicity for intermediate values and two-step periodic oscillations for large pedestrian numbers. b) Root mean square steady-state platform oscillation amplitude as a function of effective mass (equivalent to number of pedestrians. We see three different but connected trends corresponding to the three solution types in a).

**References.**

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[2] Y. Wang and M. Srinivasan, “Stepping in the direction of the fall: the next foot placement can be predicted from current upper body state in steady-state walking,” Biol. Lett., vol. 10, p. 20140405-20140405, Sep 2014